Introduction to Arithmetic and Geometric Sequences – Notes

-A sequence is simply an ordered list of numbers.
-Each number in the sequence is called a "term."
-Terms are referred to by the following notation:
-If we refer to a generic term of the sequence, we say a_n.
-The term that comes directly before a_{13} is a_{12}, so the term directly before a_n is $a_{(n-1)}$.

5,	7,	9,	11,	13,	15,…	______,	______
a_1	a_2	a_3	a_4	a_5	a_6 …	$a_{(n-1)}$	a_n

Arithmetic Sequences

Arithmetic Sequences are built by repeatedly adding the same number (called the common difference) to the first term a_1.

Arithmetic: 17, 13, 9, 5, 1, -3, -7…

$a_1 = 17$

common difference = -4

Geometric Sequences

Geometric Sequences are built by repeatedly multiplying the same number (called the common ratio) to the first term a_1.

Geometric: ¾, 3, 12, 48, 192, …

$a_1 = ¾$

common ratio = 4

<u>Recursive</u> Definition (Formula) of a Sequence

In order to describe a sequence to someone, we simply must tell them where to start, and then how to get the next term of the sequence. A recursive definition must be repeated over and over to create the sequence.

Recursive Definition of an arithmetic sequence

17, 13, 9, 5, 1, -3, -7…

Since this pattern starts with 17, we must say $a_1 = 17$.
Then to get the next number, we take the term before it, and subtract 4, (or add -4.)
So: $a_n = a_{(n-1)} - 4$

Recursive Definition of a geometric sequence

¾, 3, 12, 48, 192, …

Since this pattern starts with ¾, we must say $a_1 = ¾$.
Then to get to the next number, we take the term before it, and multiply by 4.
So: $a_n = a_{(n-1)} * 4$

Remember, recursive definitions must have <u>two parts</u>, first tell where the pattern begins, and then what math is repeated in order to get the next term of the sequence. Recursive definitions are simple, and easy to understand, and regularly used in computer programming. Unfortunately, they are tedious to use if we wanted to know a_{20}, because we would have to find a_{19} first. And to find a_{19}, we need to know a_{18}……. It gets old quickly!

Explicit Definition (Formula) of a Sequence

In order to be able to quickly find any term of a sequence, we need a more direct way, or explicit formula for each term. The explicit formula includes the first term and the math we must repeat, all in one formula.

Explicit Formula of an arithmetic sequence

17, 13, 9, 5, 1, -3, -7…

This pattern starts at 17, and then to get the 5th term (a_5), we had to add -4 how many times? Notice, to get to a_5, we only had to add -4 four times. So, the explicit formula is:

$a_n = 17 - 4(n-1)$

☞ We start with 17 and add *-4* "n-1" times.

Generically: $a_n = a_1 + d(n-1)$ where d is the common difference and n is the term we are looking for.

Explicit Definition of a geometric sequence

¾, 3, 12, 48, 192, …

This pattern starts at ¾ and then to get to the 4th term (a_4) we had to multiply by 4 three times. ("n-1" times) We use exponents to indicate repeated multiplication. So, the explicit formula is:

$a_n = \frac{3}{4} * 4^{(n-1)}$

☞ We start with ¾ and then multiply by 4 "n-1" times.

Generically, $a_n = a_1 * r^{n-1}$ where r is the common ratio and n is the term we are looking for.

So, examples from your worksheet:

$35,\ 45,\ 55,\ 65,\ \ldots$

Common Difference: 10

Recursive Formula: $a_1 = 35$

$a_n = a_{(n-1)} + 10$

Explicit Formula: $a_n = 35 + 10(n-1)$

Find a_{52}: $a_{52} = 35 + 10(52-1)$

$= 35 + 10(51)$

$= 545$

$1,\ -6,\ 36,\ -216,\ \ldots$

Common Ratio: -6

Recursive Formula: $a_1 = 1$

$a_n = a_{(n-1)} \cdot (-6)$

Explicit Formula: $a_n = 1 \cdot (-6)^{n-1}$

Find a_8: $a_8 = 1 \cdot (-6)^7$

$= -279{,}936$